Localization, Exploration, and Navigation Based on Qualitative Angle Information

Frieder Stolzenburg[1]

Abstract. Three of the major problems in qualitative spatial reasoning and robotics are localization, exploration, and navigation in known or unknown environments. This paper investigates how far different qualitative methods based on angle information, most of them originally invented for the representation of spatial knowledge only, are well-suited for these tasks. It turns out that with panoramas, which are special roundviews, the qualitative localization problem can be solved in a satisfiable manner. It is stated that the exploration problem, i.e. qualitative map building, remains difficult for all approaches. In addition, qualitative navigation operators for robot control are discussed.

1 Introduction

The main goal of qualitative spatial reasoning is to represent not only the everyday commonsense knowledge about the physical world, but also the underlying abstractions used by engineers in quantitative models (Cohn and Hazarika, 2001), in order to obtain a general model of a specific domain, as it is e.g. exemplified for robotic soccer by Dylla et al. (2005). A qualitative world model abstracts from the physical reality in order to obtain a robust and easily maintainable model. Furthermore, a qualitative model may still work, even if the exact (physical) laws are not known. Another motivation for a qualitative approach is that it is likely to be cognitively more adequate, e.g., human agents probably do not solve differential equations while interacting with their environment.

What does *qualitative* actually mean in this context? First, a qualitative representation normally is symbolic, without any continuous numerical values. The physical reality is approximated by a bounded number of states, i.e., the level of precision is decreased. Second, usually a qualitative world model yields only local information relative to the observer, i.e., the frame of reference corresponds to an egocentric point of view. Clementini et al. (1997) exemplify this e.g. for positional information. Both aspects are important for (qualitative) navigation in spatial environments: Since human and robot agents only have restricted computing resources for acting autonomously, an abstraction of the real world might be helpful. In addition, an agent has only access to local information, e.g. its internal state and a excerpt of the external world, hence it has essentially an egocentric view of the world. Following the lines of Levitt and Lawton (1990), the main questions that an agent in a spatial environment has to address are:

1. (self-)localization: where am I?
2. exploration: where are other places relative to me?
3. navigation: how do I get to other places?

In order to solve these tasks, *robot* agents may use sensors like e.g. compasses, digital video cameras, infrared sensors, laser range finders, or sonars. The corresponding numerical sensor data can be exploited by a robot. Together with probabilistic methods, imprecise or even inconsistent data can be processed (Thrun et al., 2005). However, *human* agents usually do not have access to quantitative metric information. Exact measuring of angles or distances is difficult for them. Therefore, it seems to be worthwhile to investigate qualitative approaches for localization, exploration, and navigation — not only for human but also for robot agents, whenever we are confronted with sensors, yielding very imprecise quantitative information.

In this paper, we will consider the scenario of an agent situated in a spatial environment. In order to keep things simple, we will restrict our attention mainly to two-dimensional environments or to the projection into two dimensions in this context, although most of the definitions stated here can easily be adapted to more (or less) dimensions. We assume that there is a certain number of natural or artificial landmarks in the environment, that are points without dimension, e.g. a mountain top or a beacon. Landmarks help agents to orient in known or unknown environments. A *landmark* must be a distinctive visual event, i.e., it defines a single direction, and it must be visually re-acquirable (Levitt and Lawton, 1990). Recognition of landmarks is a major problem in computer vision. We will not deal with this topic here, but suppose that, at least on a qualitative level, landmarks can be identified by the agents.

But what kind of qualitative, i.e. completely non-numerical information does an agent have available for orienting in a spatial environment? First, agents may estimate *distances* in categories like close and far. However, for humans distances are hard to measure without further remedies, and reliable distance sensors like laser range finders are nowadays still expensive. Using only odometry for (robot) navigation, i.e. tracking the distances the agent has moved among others, in order to determine the actual agent position, is often not successful. Second, agents may exploit *angle information*. Although it may be difficult to estimate angles exactly, too, qualitative angle information often is available: From a roundview, an agent may obtain (a) the (cyclic) ordering of visible landmarks. In addition, (b) left-right or other spatial relations among the landmarks (relative to the agent position) may be known. Therefore we will focus in the following on qualitative angle information (but see Sect. 7), treating both just mentioned aspects of qualitative angle information.

Let us now consider an example configuration with four landmarks, which are numbered from 1 to 4, as shown in Fig. 1. There, each of the six possible pairs of landmarks are connected by straight lines, leading to the shown tessellation, that (in this case) partitions the plane into 18 regions. By construction, all such regions are convex two-dimensional polytopes, i.e. polygons (e.g. triangles) or their unbounded counterparts. If e.g. an agent is somewhere located in region R in Fig. 1, then it sees landmark 2 left of landmark 3. We will discuss left-right relations further in Sect. 4. Furthermore, during a roundview from region R, the agent sees the landmarks in the order

[1] Hochschule Harz, Automation and Computer Sciences Department, Friedrichstr. 57-59, D-38855 Wernigerode, fstolzenburg@hs-harz.de

1234 (clockwise). Since the starting point of the roundview is not fixed, this order is cyclic and is equivalent to e.g. the order 2341. This cyclicity is problematic, as we will see in Sect. 3.

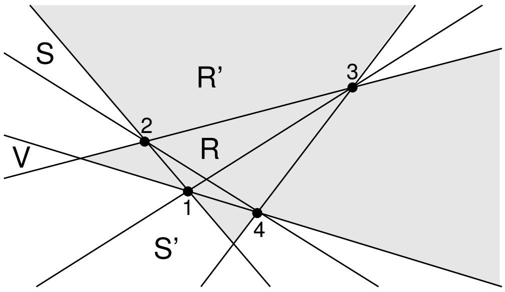

Figure 1. Example with four landmarks.

How can an agent now exploit such kind of information in qualitative localization, exploration, and navigation? This is the subject of the rest of the paper. In the next section (Sect. 2), we state more formally the basic definitions of point and line configurations based on landmarks. We discuss different means for qualitative spatial reasoning: cyclic orderings (Levitt and Lawton, 1990) and so-called aspects (Goodman and Pollack, 1993; Hübner and Wagner, 2005; Miene and Wagner, 2006) in Sect. 3, left-right relations (order types) and permutation sequences (Goodman, 1997; Goodman and Pollack, 1993) in Sect. 4, and panoramas (Schlieder, 1993, 1996) in Sect. 5. The complete set of panoramas of a configuration allows us a qualitatively exact mapping of the environment and thus addresses the exploration problem (also stated in Sect. 5). After that, qualitative operators for navigation such as crossing between two landmarks or heading toward a certain landmark are investigated (Sect. 6). Finally, we briefly compare the results stated here with related work on qualitative and also quantitative (robot) navigation (Sect. 7), including a brief excursus on distance-based configurations and position estimation. We end up with concluding remarks about qualitative spatial reasoning methods (Sect. 8).

2 Point Configurations and Line Arrangements

A *point configuration* is defined by a finite set of points P. A point configuration implicitly defines a *line arrangement*, that is the set L of all straight lines passing through pairs of points in P. Let P^* be the set of all intersection points of the straight lines in L. Clearly, it holds $P \subseteq P^*$. Note that the notion line arrangement can be defined without recursion to the notion point configuration (Goodman, 1997; Goodman and Pollack, 1993). The natural setting for line arrangements is the real projective plane. Nevertheless, we here think of them as lying in the Euclidean plane $\mathbb{R}^2$, to simplify matters. Let $n = |P|$, i.e. the number of points in P. Then, a point configuration is called *simple* iff each pair of straight lines intersects exactly once except for the points in P itself, which have multiplicity $n-1$, i.e., exactly $n-1$ lines from L are passing through them.

Each straight line of a point configuration passing through two points A and B in this direction (written as $\overline{AB}$) defines two half-spaces, namely the one to the left and the one to the right of the oriented straight line. A *region* of a configuration is a maximally connected component of the complement of the straight lines of the configuration. It can be defined as intersection of open half-spaces, i.e., a region can be identified by the set of straight lines in L, which are left or, alternatively, right of it. Tab. 1 shows the respective characterizations of the region R and R' from Fig. 1. Since both regions are immediate neighbors, their characterizations differ in only one relation: R is right of $\overline{23}$, whereas R' is left of it. Two configurations induced by the point sets P_1 and P_2, respectively, are called (qualitatively or combinatorially) *equivalent* iff they contain the same regions, where P_1 and P_2 must contain the same point identifiers.

region	left of	right of
R	$\overline{13}, \overline{14}, \overline{21}, \overline{24}, \overline{32}, \overline{43}$	$\overline{31}, \overline{41}, \overline{12}, \overline{42}, \overline{23}, \overline{34}$
R'	$\overline{13}, \overline{14}, \overline{21}, \overline{23}, \overline{24}, \overline{43}$	$\overline{31}, \overline{41}, \overline{12}, \overline{32}, \overline{42}, \overline{34}$

Table 1. Some left-right relations from Fig. 1.

Since there are $\binom{n}{2}$ pairs of landmarks and thus as many straight lines in L, in principle, $2^{\binom{n}{2}}$ different regions are possible. Obviously, not all of them occur in one configuration. In Fig. 1, there is e.g. no region that is left of $\overline{12}$, $\overline{23}$, and $\overline{31}$. But how many regions are there in a simple point configuration in general? Evidently, for $k=2$ landmarks, we have 2 regions. For $k>2$, already $\binom{k-1}{2}$ straight lines are there. A new point p_k introduces $k-1$ new straight lines. Each of them has $\binom{k-1}{2} - (k-2)$ intersection points for all but the first new straight line, which has one intersection point fewer. Each straight line leads to one plus the number of intersection points new regions. The total number of regions $\rho(n)$ can be computed as shown below.[2] Some values of $\rho(n)$ are listed in Tab. 2 together with other numbers that will be explained later on.

$$\begin{aligned}\rho(n) &= 2+\sum_{k=3}^{n}\left((k-1)\left(\binom{k-1}{2}-(k-2)+1\right)-1\right)\\ &= \frac{1}{8}n^4-\frac{3}{4}n^3+\frac{23}{8}n^2-\frac{13}{4}n+1 \;=\; O(n^4)\end{aligned}$$

n	2	3	4	5	6
$\rho(n)$	2	7	18	41	85
$(n-1)!$	1	2	6	24	120
$n!$	2	6	24	120	720
$2^{n-1}(n-1)!$	2	8	48	384	3840
$\rho^*(n)$	2	6	18	46	101

Table 2. Numbers of regions for simple configurations.

3 Localization with Cyclic Orderings and Aspects

If quantitative data is not available, clearly the exact positions of landmarks cannot be computed. Only a qualitative map of the environment (i.e. the corresponding collection of regions) can be constructed by the agent. In particular, exact self-localization is not possible, if only qualitative angle information is available. Nevertheless, it should be possible to determine the region in which the agent is, where different regions should be represented by distinct identifiers. For this purpose, we will investigate roundviews at first, as done by Levitt and Lawton (1990) among others, that can be obtained by so-called omnivision cameras, allowing a view angle of 360° in total. Such cameras are used e.g. in RoboCup robotic soccer competitions. If we assume that the agent is never located exactly on one of the straight lines in L, then a *qualitative roundview* of the agent in clockwise order gives us the cyclic ordering of the n given, visible landmarks. It is characterized by a sequence of the n landmark identifiers. Because of the cyclicity, the first landmark in this sequence can be chosen arbitrarily. Therefore with the roundview approach, only $(n-1)!$ regions can be distinguished.

However, Tab. 2 reveals that this number is smaller than the number of regions $\rho(n)$ for $n \leq 5$. Hence, there must be regions in Fig. 1, which have identical cyclic orderings. In fact, this is the case e.g. for

[2] The number of (orientation) regions is given without proof in Levitt and Lawton (1990).

the regions R and R'. More generally, when crossing a connecting line between any two landmarks, then the cyclic ordering does not change. Hence, all shaded regions in Fig. 1 are characterized by one and the same cyclic ordering 1234. Even worse, for $n \geq 4$, it might be the case that regions, which are not even immediate neighbors, are associated with the same cyclic ordering. For the example in Fig. 1, this holds e.g. for the regions S and S' with associated cyclic orderings 3421 and 2134, respectively, which are identical.

It turns out, that the problem with cyclic orderings is their cyclicity. Alternatively, regions can be identified uniquely by means of what we call aspects (Goodman and Pollack, 1993), if we restrict ourselves to points in regions outside the convex hull of all landmarks. In this case, an *aspect* is defined by the permutation of the n landmarks in the order from left to right. For this, we first must define the *left-right relation* (called orientation of landmark pair boundaries by Levitt and Lawton, 1990) more precisely: landmark A is left of landmark B iff the azimuth (clockwise oriented) angle $\measuredangle(A,B)$ seen from the current point of view (agent position or region) is between $0°$ and $180°$ (exclusively). Hence, in order to measure a left-right relation, we must be able to detect $180°$ angles by means of sensors. By using a camera with view angle equal or smaller than $180°$, this is certainly possible.

Since for all points in regions outside the convex hull, there always is a leftmost and rightmost visible landmark, thus aspects are in contrast to simple roundviews non-cyclic. Because of this, up to $n!$ aspects can be distinguished, which is always greater than $\rho(n)$ (see Tab. 2). Fig. 2 shows for the running example, that all regions outside the convex hull (shaded) have different aspects. This holds in general for any configuration, because, for regions outside the convex hull, the left-right relation is a linear order and can easily be read off the aspects: landmark A is left of landmark B or, stated differently, the point of view is right of $\overline{AB}$ iff A occurs before B in the respective permutation. Hence, each aspect uniquely determines a certain intersection of open half-spaces, i.e. a region.

In the context of the RoboCup robotic soccer scenario, aspects have been applied successfully in a case study with four-legged robots (Hübner and Wagner, 2005; Miene and Wagner, 2006). There, the robots move around the goal area and try to localize themselves.

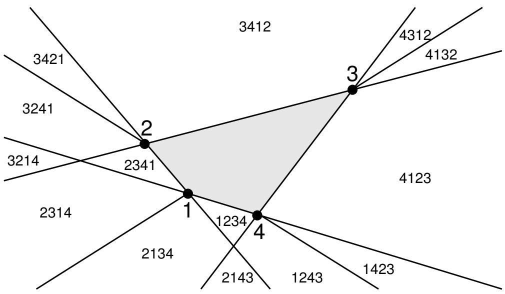

Figure 2. Aspects of regions outside the convex hull.

4 Order Types and Permutation Sequences

Cyclic orderings and aspects exploit the left-right relations between landmarks seen from a point of view from within a region. If we collect the order information of all triples of landmarks (i.e. allowing landmarks as points of view so to speak), we get the so-called *order type* of a configuration. We say, the triangle of landmarks ABC is ordered positively iff C is left of $\overline{AB}$, i.e., A, B, and C are oriented counter-clockwise. This can easily be generalized to any number of spatial dimensions $d \geq 1$ by means of determinants (see e.g. Goodman and Pollack, 1983): a sequence $p_0 \cdots p_d$ of $d+1$ points in $\mathbb{R}^{\mathrm{d}}$ with $p_i = (x_{i1}, \cdots, x_{id}, 1)$ for $0 \leq i \leq d$ has *positive orientation* iff $\det(x_{ij}) > 0$.

Unfortunately, there are non-equivalent configurations which have the same order type. Thus in general, order types do not uniquely determine a configuration qualitatively. In order to see this, look at the configuration in Fig. 3, that shows the reflection of the one in Fig. 1, where the landmarks are numbered similarly in both cases. Both configurations have the same order type, and they consist of the same regions except for the regions V and V', respectively. Therefore in summary, it is not sufficient to get the left-right relations for the n kernel points (the landmarks), in order to identify a configuration qualitatively.

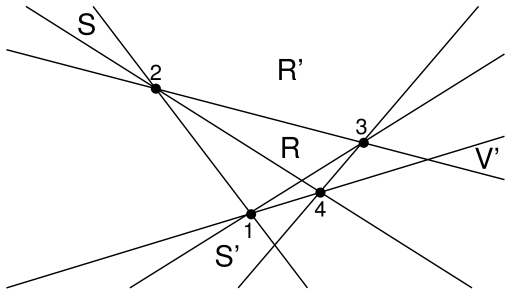

Figure 3. Reflection of the example in Fig. 1.

In order to overcome this problem, the notion permutation sequence of a configuration has been introduced in the literature (Goodman, 1997; Goodman and Pollack, 1993). For this, we project all landmark points orthogonally onto a straight reference line r, e.g. one of the two coordinate axes, thus again obtaining a permutation of the landmarks determined by the order in which the points fall onto r. Let now r rotate counter-clockwise. Then a new permutation arises whenever r passes through a direction orthogonal to one of the connecting lines $l \in L$, say $\overline{AB}$. If the landmarks A and B appear in the induced permutation in that order before r passed orthogonally to l, both landmarks will appear in reverse order in the permutation induced on r just after. The reversal of the landmarks in the *move* from one permutation to the next is called *switch*.[3] Allowing r to continue rotation, we obtain the circular, doubly infinite *permutation sequence* associated with the given configuration. This sequence is clearly periodic —the period corresponds to a full rotation of r—, and is in fact determined by a half period (Goodman and Pollack, 1993). Fig. 4 illustrates these definitions for the configuration of Fig. 1. Starting with the x-axis as rotating line r, we get the permutation 2143. After three moves with the switched lines $\overline{12}$, $\overline{42}$, and then $\overline{41}$, rotating r counter-clockwise by $90°$, we arrive at the permutation 4123, which is the projection onto the y-axis.

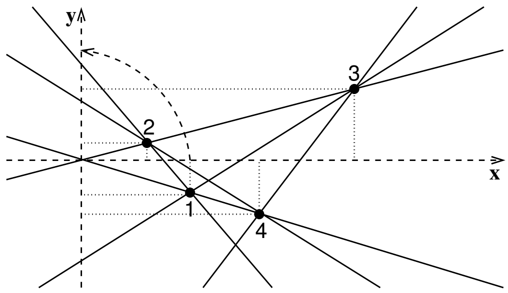

Figure 4. Projections on a straight line, rotating counter-clockwise.

Incidentally, the permutation sequences for the configurations in Fig. 4 and its reflection in Fig. 3 are different: in the original con-

[3] Note that we restrict attention to simple point configurations in this context.

figuration (Fig. 1 or 4) the switch wrt. $\overline{41}$ occurs just before the one wrt. $\overline{32}$, whereas this is the other way round in its reflection (Fig. 3). However, if we only consider the permutation sequences wrt. the given n landmarks, not taking into account the additional intersection points (as we have done so far), then the configuration is not always completely determined by its permutation sequence. To see this, look at the example with $n = 6$ landmarks, sketched in Fig. 5. There, the lines $\overline{12}$, $\overline{34}$, and $\overline{56}$ intersect in one point (namely where the shaded regions T and T' touch each other). However, slightly moving landmark 3 up or down, respectively, leads to configurations which have identical permutation sequences, although they are not equivalent, because one contains region T and the other region T' which are different. Therefore, the application of permutation sequences is of limited use, too. In addition, it remains an open practical question, how intersection points, orthogonal projections, let alone complete permutation sequences can be recorded by any concrete sensors. We have the same problems concerning symbolic projection (Schlieder, 1996) where only two qualitative projections on orthogonal axes as in architecture are considered.

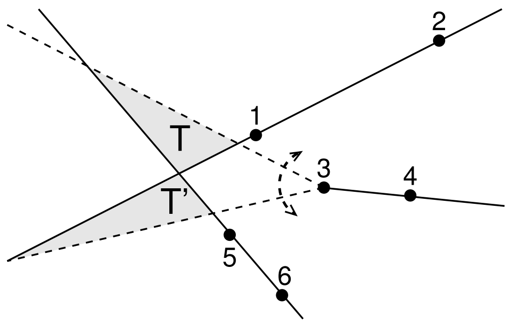

Figure 5. Problematic example for permutation sequences.

5 Panoramas and the Exploration Problem

From Sect. 4, we may conclude that a configuration can only be identified wrt. qualitative equivalence, if all regions contained in it are known. However, neither cyclic orderings nor aspects (see Sect. 3) allow us to name regions in a unique manner. To solve this problem, panoramas are introduced by Schlieder (1993, 1996): for the landmark points $p \in P$ and the point of view s, the clockwise oriented cyclic ordering of the $2n$ straight lines of the form $\overline{ps}$ and $\overline{sp}$ is called *panorama* of s. It corresponds to the order in which an agent sees the landmarks and their backs (although it is not easy to imagine how backs can be detected by concrete sensors, unless 180° angles can be measured). Therefore, a straight line of the form $\overline{ps}$ is abbreviated by $\overline{p}$, while $\overline{sp}$ is simply written as p.

Since panoramas have a period of length $2n$ where the second half is just the reverse of the first half, a convenient notation of a panorama is given by stating only the first half of a period starting with an arbitrary landmark or its back. Hence, up to $2^{n-1}(n-1)!$ panoramas can be distinguished, which is always greater than $\rho(n)$ (see Tab. 2). A panorama always defines a unique region in a configuration, because it determines all left-right relations as follows: a point s is left of the line $\overline{pq}$ iff $pq\overline{p}\overline{q}$ occurs in the panorama of s. For the example configuration in Fig. 1, the panorama of region R is $1\overline{3}\overline{2}\overline{4}\overline{1}3\overline{2}4$, that of R' is $12\overline{3}\overline{4}\overline{1}\overline{2}34$. In both cases, stating only the first half would suffice. Note that deleting the backs $\overline{p}$ of all landmarks in the panoramas yields us the ordinary cyclic ordering of the respective region (here: 1234 in both cases).

In summary, panoramas finally solve the localization problem (see Sect. 1), because a unique identification of regions is possible with them. Can they also solve the exploration problem? The answer is yes, but in order to build a map of an unknown environment, a robot agent has to explore, i.e. to visit many regions. We have seen in Sect. 4, that the order type does not completely specify the configuration. In addition, the example in Fig. 2 teaches us that it is not sufficient to determine the aspects of some regions around the convex hull. For this example (see also Fig. 1) it is necessary to find out whether region V is contained in it or not, in order to distinguish this configuration from the one in Fig. 3 with the region V'. The example can easily be generalized by repeatedly putting one of both configurations (Fig. 1 or Fig. 3) on top of the other. Then at least all of the copies of the regions V or V', respectively, have to be visited, i.e. at least $\frac{1}{18}n$. Hence, the exploration problem, i.e. qualitative map building, turns out to be very expensive, since the number of regions to be visited cannot be bounded by a constant or a logarithmic function, as the example shows.

This negative result for qualitative exploration seems to be related to the stretchability problem in discrete geometry (Goodman, 1997; Goodman and Pollack, 1993). There, so-called pseudoline arrangements are considered. A *pseudoline* is a simple curve in the plane going to infinity in two directions, where any two members intersect each other at most once, and cross if they intersect. Like point configurations, a pseudoline arrangement induces a set of regions in the plane. It is called *stretchable* or realizable iff there is a straight line arrangement with the same combinatorial structure. This problems turns out to be NP-hard (Shor, 1991). If a configuration could be determined by knowing only a few regions of the configuration, the realizability problem could be answered efficiently, too.

As we have just seen, the exploration problem is difficult, if only qualitative information is available. With *quantitative* information, however, this appears to be different: in most cases, localization and map building is possible by making only a few snapshots of the environment. First, if e.g. the agent is able to measure distances and absolute orientations, i.e. wrt. to a global reference coordinate system, clearly the snapshot from only one point position is sufficient to localize the agent or one of the landmarks. Second, if the agent can measure absolute orientations only, e.g. by a compass, then two snapshots from known positions are enough for computing an unknown landmark position. Third, if only distance information is available, three or sometimes even two snapshots suffice (Levitt and Lawton, 1990). Last but not least, Betke and Gurvits (1997) describe a method for localizing a mobile robot in an environment with landmarks, where only their bearings relative to each other are given (i.e. relative angle information). Given such possibly noisy input, the algorithm estimates the robot position and orientation with respect to the map of the environment. The algorithm makes use of complex numbers and runs in time linear in the number of landmarks, employing a least squares approach.

6 Navigation

After having discussed the qualitative localization and exploration problem, let us now come to the navigation problem: how do I get to other places? If quantitative information is available and there are no obstacles around, the shortest way from one place to another obviously is simply traveling along the straight line segment connecting both places. Otherwise, exact quantitative distances cannot be measured. Then, a qualitative notion of distance is needed. Since qualitative localization cannot be more precise than determining the region the agent is in, a natural definition of qualitative distance is the number of regions that have to be passed (minimally) while going from one place to the other.

Formally, the *qualitative distance* between two regions can be defined as the number of point pairs inverted in their respective panoramas. Consider the regions X and X' in Fig. 6, whose respective panoramas are $2134\overline{2134}$ and $3241\overline{3241}$, respectively. The first panorama can be transferred into the second one by three inversions, namely 13, 23, and 14, since the corresponding lines $\overline{13}$, $\overline{23}$, and $\overline{14}$ must be crossed. Thus, the qualitative distance between the regions X and X' is 3. From this observation, Schlieder (1993) proposes a greedy algorithm for finding shortest qualitative paths. For this, the environment, i.e. the configuration has to be known in advance, however. In each step, a straight line of L is crossed, which is (still) inverted in the current panorama compared with the goal panorama. This simple greedy procedure solves the qualitative navigation problem efficiently. It is complete and optimal, i.e. it always finds a shortest path wrt. qualitative distance, provided that there are no obstacles in the way. In the latter case, a more general search procedure like A* (see e.g. Russell and Norvig, 1995), that is still optimal and complete, can be applied. In Fig. 6 one can see, that there are two possible shortest path from X to X' in the example: XUVX' and XUWX'. Note that the order in which the lines have to be crossed is not completely free.

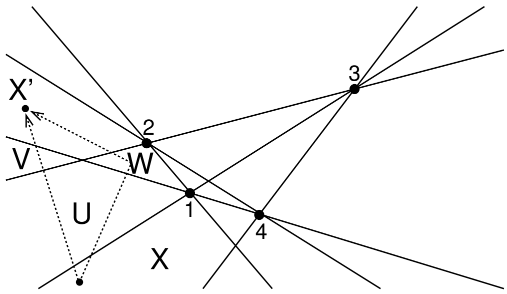

Figure 6. Finding the qualitatively shortest path.

Now the question remains, how can we come from one region to a neighbored one by means of qualitative operators. Schlieder (1993) does not answer this question, nevertheless he provides a clear representation of qualitative navigation. Levitt and Lawton (1990) propose several qualitative operators that could be used not only for cognitively adequate description (localization) but for controlling (robot) agent behavior, e.g. $at(p)$ (head toward landmark p), $btw(p_1,p_2)$ (crossing between landmarks p_1 and p_2 along the angular bisector, which leads to a hyperbolic trajectory), or $left(p_1,p_2)$ (crossing $\overline{p_1p_2}$ left of p_1). All operators are executed, until a straight line of L is reached. In the example, the operator sequence $at(2,1)$ $btw(2,1)$ $left(2,1)$ would do the job. Unfortunately, it is not possible to determine the appropriate operator from the actual panorama alone, unless the complete configuration is known. Therefore, we will not go into more details here.

7 Related Works

There are numerous related works in the field of qualitative spatial reasoning (Cohn and Hazarika, 2001), addressing the representation problem of configurations in the plane, exploiting only completely non-numerical information. Latecki and Röhrig (1993) e.g. start with the observation that humans are able to recognize the right angle, and so are able to distinguish an acute from an obtuse angle (i.e. less or more than 90°, respectively). They associate a triangle ABC with an ordered pair consisting of its triangle orientation (clockwise or counter-clockwise) and the qualitative angle at B (acute or obtuse). Although augmented configuration knowledge with this kind of qualitative angles allows a finer subdivision of the plane, it still does not enable us to distinguish the configuration in Fig. 1 from its reflection in Fig. 3, that are not qualitatively equivalent.

The CYCORD approach, based on the clockwise order of directions of straight lines (closely related to permutation sequences, see Sect. 4), allows the unified treatment of several qualitative spatial calculi (Röhrig, 1997). Its reasoning system finds out whether a set of orientations in the plane (called *constraints* in this context) is consistent (i.e. realizable in the plane), which however is NP-complete in general. A refinement of the theory (Isli and Cohn, 1998) makes not only use of the relations *left* and *right* between two directions, but also of *equal* and *opposite*. These four binary atomic relations lead to 24 ternary relations for cyclic ordering of directions in the plane, hence 2^{24} relations in total, namely including all possible unions of ternary relations. Isli and Cohn (1998) identify a subclass of the theory, whose constraint problem is tractable. Nonetheless, with this approach, a distinction of the two configurations sketched in Fig. 5 is not possible.

So far, we concentrated on exploiting only completely qualitative information, like left-right relations or ordering information. If more, yet noisy quantitative data is available, then more sophisticated procedures are possible. In Busquets et al. (2003) e.g., a multiagent approach to qualitative landmark-based navigation with triangulation is presented. Frommberger (2006) considers a simple goal-directed navigation task, where a robot agent must find a specific landmark in an unknown environment, avoiding collisions with obstacles, and proposes a solution that employs reinforcement learning. Yairi and Hori (2003) introduce a map learning method for mobile robots, employing probabilities for the co-visibility of objects. This approach makes use of the assumption that a pair of objects observed at the same time is likely to be located more closely together than others for which this is not the case. Furthermore, there are several, very successful probabilistic methods applied to what is called the simultaneous localization and map building (SLAM) problem in robotics (Thrun et al., 2005). Other methods solve the calibration problem for video cameras and allow to detect objects from photographs (photogrammetry) (Tsai, 1986).

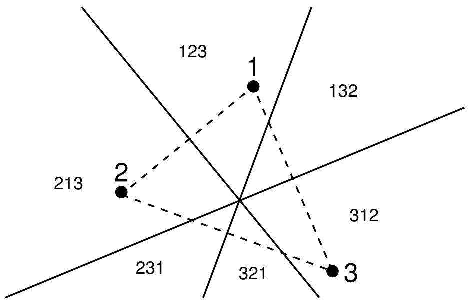

Figure 7. Regions induced by the perpendicular bisectors of all lines.

Dual to the approach followed in this paper, we may take qualitative distances into closer consideration instead of angle information. Then, from a set of n landmark points, again a configuration of regions is induced, this time by the perpendicular bisectors of the sides connecting all pairs of points. In this context, each region can uniquely be identified by one of the $n!$ orders of landmarks in increasing distance. This concept can easily be generalized to higher dimensions. Fig. 7 shows an example in the plane for $n = 3$ with six regions. Although distance-based configurations cannot be distinguished from its reflections, they may well be applied in practice, namely in robot localization based on the signal strength in wireless computer networks (WLAN) (Grabe, 2007; Ibach et al., 2004). Since the signal strength is very noisy, a qualitative approach may be a good idea there.

As in the angle-based setting, few landmarks already yield a fine tessellation of the plane. For each of the $\binom{n}{3}$ possible triangles of the given points, the three corresponding perpendicular bisectors of the side intersect in one point, namely in the center of the respective circumscribed circle. Because of this, we have $\binom{n}{3}$ regions less than in simple line arrangements with N lines in the plane. According to Graham et al. (1994), N lines lead to $\frac{N(N+1)}{2}+1$ regions in a simple line arrangement, where always exactly two lines intersect in one point. Since we have $N=\binom{n}{2}$ perpendicular bisectors of the sides, as the number of distance-based regions $\rho^*(n)$ we get the following formula (see also Tab. 2):

$$\begin{aligned}\rho^*(n) &= \left(\frac{N(N+1)}{2}+1\right)-\binom{n}{3}\\ &= \frac{1}{8}n^4-\frac{5}{12}n^3+\frac{7}{8}n^2-\frac{7}{12}n+1 \;=\; O(n^4)\end{aligned}$$

8 Conclusions and Future Work

In this paper, we discussed how far different qualitative spatial representation methods are suited to solve the localization, exploration, and navigation problem. It turns out that panoramas allow to solve the qualitative localization problem adequately. As shown in this paper, the qualitative and discrete exploration problem is difficult, compared with its quantitative counterpart, because always a certain fraction of regions has to be visited which cannot be bounded by a constant or a logarithmic function. Finally, qualitative navigation operators for robot control are discussed. In future work, it should be investigated how qualitative and quantitative spatial methods for localization, exploration, and navigation can benefit more from each other, e.g. in the context of photogrammetry, i.e. recognizing objects and positions from several snapshots in more than two dimensions.

REFERENCES

M. Betke and L. Gurvits. Mobile robot localization using landmarks. *IEEE Transactions on Robotics and Automation*, 13(2):251–263, 1997.

D. Busquets, C. Sierra, and R. L. de Màntaras. A multiagent approach to qualitative landmark-based navigation. *Autonomous Robots*, 15(2):129–154, 2003.

E. Clementini, P. Di Felice, and D. Hernández. Qualitative representation of positional information. *Artificial Intelligence*, 95(2): 317–356, 1997.

A. G. Cohn and S. M. Hazarika. Qualitative spatial representation and reasoning: An overview. *Fundamenta Informaticae*, 43(1): 2–32, 2001.

F. Dylla, A. Ferrein, G. Lakemeyer, J. Murray, O. Obst, T. Röfer, F. Stolzenburg, U. Visser, and T. Wagner. Towards a league-independent qualitative soccer theory for RoboCup. In D. Nardi, M. Riedmiller, C. Sammut, and J. Santos-Victor, editors, *RoboCup 2004: Proceedings of International RoboCup Symposium*, LNAI 3276, pages 611–618. Springer, Berlin, Heidelberg, New York, 2005.

L. Frommberger. A qualitative representation of structural spatial knowledge for robot navigation with reinforcement learning. In *Proceedings of Workshop on Structural Knowledge Transfer for Machine Learning*, Pittsburgh, PA, 2006.

J. E. Goodman. Pseudoline arrangements. In J. E. Goodman and J. O'Rourke, editors, *Handbook of Discrete and Computational Geometry*, chapter 5, pages 83–109. CRC Press, Boca Raton, New York, 1997.

J. E. Goodman and R. Pollack. Multidimensional sorting. *SIAM Journal on Computing*, 12(3):484–507, 1983.

J. E. Goodman and R. Pollack. Allowable sequences and order types in discrete and computational geometry. In J. Pach, editor, *New Trends in Discrete and Computational Geometry*, chapter 5, pages 103–134. Springer, Berlin, Heidelberg, New York, 1993.

M. Grabe. Qualitative Distanz-basierte Positionsbestimmung. Diplomarbeit, Fachbereich Automatisierung und Informatik, Hochschule Harz, 2007. To appear.

R. L. Graham, D. E. Knuth, and O. Patashnik. *Concrete Mathematics*. Addison-Wesley, Reading, MA, 2nd edition, 1994.

K. Hübner and T. Wagner. An egocentric qualitative spatial knowledge representation based on ordering information for physical robot navigation. In D. Nardi, M. Riedmiller, C. Sammut, and J. Santos-Victor, editors, *RoboCup 2004: Proceedings of International RoboCup Symposium*, LNAI 3276, pages 134–149. Springer, Berlin, Heidelberg, New York, 2005.

P. Ibach, T. Hübner, and M. Schweigert. MagicMap – kooperative Positionsbestimmung über WLAN. In *Chaos Communication Congress Proceedings*, Berlin, 2004.

A. Isli and A. G. Cohn. An algebra for cyclic ordering of 2D orientation. In *Proceedings of 15th American Conference on Artificial Intelligence*, pages 643–649, Madison, WI, 1998. AAAI/MIT Press.

L. J. Latecki and R. Röhrig. Orientation and qualitative angle for spatial reasoning. In *Proceedings of 13th International Joint Conference on Artificial Intelligence*, pages 1544–1549, Chambéry, France, 1993. IJCAI Inc., San Mateo, CA, Morgan Kaufmann, Los Altos, CA. Volume 1.

T. S. Levitt and D. T. Lawton. Qualitative navigation for mobile robots. *Artificial Intelligence*, 44(3):305–360, 1990.

A. Miene and T. Wagner. Static and dynamic qualitative spatial knowledge representation for physical domains. *KI*, 2/06:30–35, 2006.

R. Röhrig. Representation and processing of qualitative orientation knowledge. In G. Brewka, C. Habel, and B. Nebel, editors, *KI-97: Advances in Artificial Intelligence – Proceedings of the 21st Annual German Conference on Artificial Intelligence*, LNAI 1303, pages 219–230, Freiburg, 1997. Springer, Berlin, Heidelberg, New York.

S. Russell and P. Norvig. *Artificial Intelligence – A Modern Approach*. Prentice Hall, Englewood Cliffs, NJ, 1995.

C. Schlieder. Representing visible locations for qualitative navigation. In N. Piera Carrete and M. G. Singh, editors, *Qualitative Reasoning and Decision Technologies*, pages 523–532. CIMNE, Barcelona, 1993.

C. Schlieder. Ordering information and symbolic projection. In S. K. Chang, E. Jungert, and G. Tortora, editors, *Intelligent Image Database Systems*, pages 115–140. World Scientific, Singapore, 1996.

P. W. Shor. Stretchability of pseudolines is NP-hard. In P. Gritzman and B. Sturmfels, editors, *Applied Geometry and Discrete Mathematics – The Victor Klee Festschrift*, DIMACS Series in Discrete Mathematics and Theoretical Computer Science 4, pages 531–554. American Mathematical Society, Providence, RI, 1991.

S. Thrun, W. Burgard, and D. Fox. *Probabilistic Robotics*. MIT Press, Cambridge, MA, London, 2005.

R. Y. Tsai. A versatile camera calibration technique for high accuracy 3D machine vision metrology using off-the-shelf TV cameras and lenses. *IEEE Journal of Robotics and Automation*, 3(4):323–344, 1986.

T. Yairi and K. Hori. Qualitative map learning based on co-visibility of objects. In *Proceedings of 18th International Joint Conference on Artificial Intelligence*, pages 183–188, 2003.